

**UNCLASSIFIED**

**AD 414133**

**DEFENSE DOCUMENTATION CENTER**

**FOR**

**SCIENTIFIC AND TECHNICAL INFORMATION**

**CAMERON STATION, ALEXANDRIA, VIRGINIA**



**UNCLASSIFIED**

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

AFOSR 4933

6 - 4 - 5

4133

AD

AF EOAR GRANT 63 - 3

20th MAY, 1963.

CATALOGED BY DDC

AS AD No. \_\_\_\_\_

414133

TECHNICAL NOTE NO. 13.

PION-NUCLEON SCATTERING AND THE J = 2, T = 0

PION-PION INTERACTION

G. C. Oades

Department of Physics,  
University College, Gower Street, London W.C.1.

The research reported in this document has been sponsored in part  
by,

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH, OAR  
through the European Office, Aerospace Research, United States  
Air Force.

PION-NUCLEON SCATTERING AND THE  $J = 2$ ,  
 $T = 0$  PION-PION INTERACTION(\*)

---

G. C OADES  
DEPARTMENT OF PHYSICS.

UNIVERSITY COLLEGE LONDON, GOWER STREET,  
LONDON W. C. 1.

MAY, 1963.

(\*) This work has been supported in part by the Air Office of Scientific Research, O.A.R. (European Office, Aerospace Research, U.S.A.F. )

ABSTRACT.

The effect of the  $J = 2$ ,  $T = 0 \pi\text{-}\pi$  interaction on the  $\pi\text{-}N$  invariant amplitude  $B^{(+)}$  is analysed. It is found that the  $\pi\text{-}N$  scattering data is inconsistent with a  $J = 2$ ,  $T = 0 \pi\text{-}\pi$  phase,  $\delta_2^0$ , which rises to above  $13^\circ$  around 650 MeV. The data is consistent with a  $\delta$ -function contribution at 1200 MeV but it is impossible to say whether this corresponds to a resonant phase or only a sharp peak in the corresponding absorptive part of the amplitude.

## INTRODUCTION

In the  $\pi\pi \rightarrow N + \bar{N}$  channel of the pion-nucleon system only states with isospin  $T = 0$  and angular momentum  $J \geq 2$  contribute to the pion-nucleon total invariant amplitude  $B^{(+)}$ . Since this amplitude affords a means of investigating the  $J = 2$ ,  $T = 0$   $\pi\pi$  interaction without interference from the  $J = 0$ ,  $T = 0$  state it is of interest to consider possible methods of studying  $B^{(+)}$ .

The  $\pi - N$  total invariant amplitudes,  $A_{\pm}^{(\pm)}$  and  $B_{\pm}^{(\pm)}$  have been studied at fixed angles in both the forward and backward directions<sup>(1), (2)</sup> since no difficulties due to divergences of Legendre series are encountered in these cases. It is necessary, however, to approximate unitarity by retaining only a small number of terms in the partial wave expansions of the amplitudes. The resulting errors may be considerable if the convergence of these series is slow, as is to be expected if there are appreciable low energy  $\pi\pi$  effects. Accordingly it is of interest to consider the amplitudes formed by integrating the total amplitudes over all physical angles. These amplitudes have distant singularities which cannot be calculated in terms of convergent Legendre series but have the advantage that the contributions of alternate terms of the partial wave expansion are much reduced in the low energy physical region. Thus it is possible to calculate nearby singularities more accurately than in the fixed angle case at the expense of introducing distant singularities which must be represented by some approximation scheme.

Hence a dispersion relation is written for the amplitude formed by integrating  $B^{(+)}$  over all angles and the results are analysed by methods similar to those which have been successfully applied to the analysis of  $\pi - N$  partial

---

(1) J. Hamilton and W. S. Woolcock, U. C. L. Physics Dept. preprint 1962.

- This paper gives a detailed review of  $\pi - N$  dispersion relations in the forward direction.
- (2) D. Atkinson, Phys. Rev. 128, 1908 (1962).

2.

waves <sup>(3)</sup> so as to give values for  $\delta_2^0$ , the  $J = 2, T = 0 \pi\cdot\pi$  phase. The dispersion relation is described in Sec. 2; the contribution from the  $\pi\pi \rightarrow N\bar{N}$  channel and its relation to the  $J = 2, T = 0 \pi\pi$  interaction are considered in Sec. 3, and the analysis of the results in terms of the phase  $\delta_2^0$  is discussed in Sec. 4.

## 2. The Discrepancy.

### (i) Kinematics

The notation follows the standard usage. The total amplitude with isospin  $T$  is given by

$$B^{(T)}(s, x) = 8\pi W \left[ \frac{f_1^{(T)}(s, x)}{(W+M)^2 - \mu^2} + \frac{f_2^{(T)}(s, x)}{(W-M)^2 - \mu^2} \right], \quad (1)$$

where  $f_1^{(T)}$  and  $f_2^{(T)}$  are expressable in terms of partial wave expansions

$$f_1^{(T)}(s, x) = \sum_{l=0}^{\infty} f_{l+}^{(T)}(s) P_{l+1}'(x) - \sum_{l=2}^{\infty} f_{l-}^{(T)}(s) P_{l-1}'(x) \quad (2)$$

$$f_2^{(T)}(s, x) = \sum_{l=1}^{\infty} \left( f_{l-}^{(T)}(s) - f_{l+}^{(T)}(s) \right) P_l'(x) \quad (3)$$

Here  $M$  is the mass of the nucleon,  $\mu$  is the mass of the pion,  $W = s^{\frac{1}{2}}$ , where  $s$  is the square of the total energy in the centre of momentum system and  $x$  is the cosine of the scattering angle in the same system.  $f_{l\pm}^{(T)}$  are the usual partial wave amplitudes and the dash denotes differentiation with

---

(3) J. Hamilton, P. Menotti, G. C. Oades and L. L. J. Vick, Phys. Rev. 128  
1881 (1962).

---

respect to  $x$ . It is also convenient to introduce the other Mandelstram variables  $t$  and  $u$ , the squares of the total energies in the other two channels.

In the following the units are chosen so that  $\mu = \hbar = c = 1$

The amplitude  $B^{(+)}$ , defined by

$$B^{(+)} = \frac{1}{3} B^{(1/2)} + \frac{2}{3} B^{(3/2)}, \quad (4)$$

is used to form the amplitude  $B_0^{(+)}$

$$B_0^{(+)}(s) = \frac{1}{2} \int_{-1}^{+1} dx B^{(+)}(s, x) P_0(x). \quad (5)$$

Substituting (1) into (5) leads to the partial wave expansion

$$B_0^{(+)}(s) = \frac{8\pi W}{(W+M)^2 - 1} \sum_{\substack{l=0 \\ \text{even } l}}^{\infty} (f_{l+}^{(+)} - f_{l-}^{(+)}) + \frac{8\pi W}{(W-M)^2 - 1} \sum_{\substack{l=1 \\ \text{odd } l}}^{\infty} (f_{l-}^{(+)} - f_{l+}^{(+)}) , \quad (6)$$

where  $f_{l\pm}^{(+)}$  are defined in terms of  $f_{l\pm}^{(+)}$  by a relation similar to (4). It will be seen that the even  $l$  terms are damped by a factor  $O(1/M^2)$  in the low energy region compared to the odd  $l$  terms thus leading to the better convergence noted in Sec. 1.

### (ii) The Dispersion Relation.

The singularities of the function  $B^{(+)}(s, t)$  are given by its double spectral representation,

$$\begin{aligned} B^{(+)}(s, t) &= G_R^2 \left( \frac{1}{u-M^2} - \frac{1}{s-M^2} \right) + \frac{1}{\pi^2} \int_{(M+1)^2}^{\infty} ds' \int_u^{\infty} dt' \frac{\rho_{12}(s', t')}{(s'-s)(t'-t)} \\ &+ \frac{1}{\pi^2} \int_{(M+N)^2}^{\infty} ds' \int_u^{\infty} du' \frac{\rho_{13}(s', u')}{(s'-s)(u'-u)} + \frac{1}{\pi^2} \int_u^{\infty} dt' \int_u^{\infty} du' \frac{\rho_{23}(t', u')}{(t'-t)(u'-u)}, \end{aligned} \quad (?)$$

$G_R$  being the rationalised pseudoscalar coupling constant.

The singularities of  $B_o^{(+)}(s)$  are determined in an analogous fashion to those of the partial wave amplitudes<sup>(4)</sup>. They are of the same form apart from the fact that there is no irrationality cut; they are shown in Fig. 1. Thus a dispersion relation for  $B_o^{(+)}(s)$  can be written in the form

$$\operatorname{Re} B_o^{(+)}(s) = \frac{1}{\pi} \int_{(M+1)^2}^{\infty} ds' \frac{\operatorname{Im} B_o^{(+)}(s')}{s'-s} + \frac{1}{\pi} \int_0^{(M-1)^2} ds' \frac{\operatorname{Im} B_o^{(+)}(s')}{s'-s} + G_B^{(+)}(s) + \Delta_B^{(+)}(s), \quad (8)$$

where either the first or the second integral is to be evaluated as a principal value integral according to whether  $s > (M+1)^2$  or  $0 < s \leq (M-1)^2$ . Here

$G_B^{(+)}(s)$ , given by

$$G_B^{(+)}(s) = - \frac{G_R^2}{s - M^2} + \frac{1}{\pi} \int_{(M-1/M)^2}^{M^2+2} ds' \frac{\operatorname{Im} B_o^{(+)}(s')}{s'-s}, \quad (9)$$

represents the contribution of the direct Born term and of the long range crossed Born term. The discrepancy  $\Delta_B^{(+)}(s)$ , contains the contributions from the circular cut,  $|s|=M^2-1$ , and the left hand cut  $-\infty < s \leq 0$ .

### (iii) High Energy Behaviour.

In writing (8) in this form it has been assumed that no subtractions are needed. The behaviour of  $B_o^{(+)}(s)$  for large  $s$  can be studied in the approximation of a finite range interaction. If the range of interaction is  $R$ , scattering is expected to become negligible for  $l \geq Rq$ , where  $q$  is the momentum in the centre of momentum system. Then from (6),

$$B_o^{(+)}(s) \underset{s \text{ large}}{\sim} \frac{8\pi}{s^{1/2}} \sum_{l=0}^{Rq} (-1)^l (f_{l+}^{(+)} - f_{l-}^{(+)}). \quad (10)$$

(4) J. Hamilton and T. D. Spearman. Ann. Phys. 12, 172 (1961).

Also since unitarity ensures that

$$|f_{l\pm}^{(\pi)}| \leq 1/q ,$$

an upper bound is obtained for  $B_o^{(+)}(s)$  of the form

$$|B_o^{(+)}(s)| \lesssim \frac{8\pi}{s^{1/2}} \frac{2Rq+1}{q} \sim \frac{16\pi R}{s^{1/2}} . \quad (11)$$

Even if, as has been suggested by Regge pole theories,  $R$  increases logarithmically with energy (11), together with a theorem by Sugawara and Karazawa (5) are sufficient to ensure that (8) is well defined without having to introduce a subtraction.

In the case of the series for  $\text{Im } B_o^{(+)}(s)$  since the contribution from each value of  $l$  is of the form  $(-1)^l (\text{Im } f_{l+}^{(+)} - \text{Im } f_{l-}^{(+)})$  and since  $\text{Im } f_{l\pm}^{(+)} > 0$ , the high energy value of  $\text{Im } B_o^{(+)}(s)$  is strongly dependent on forces of the spin-orbit type and it is very probable that it falls off more quickly than suggested by (11). It is assumed in the calculations that

$$0 < |\text{Im } B_o^{(+)}(s)| \leq 16\pi/s^{1/2} \quad (12)$$

above 2 BeV.

(iv) Evaluation of the Discrepancy.

In (8) it is possible to calculate all terms, apart from the discrepancy, using physical pion-nucleon data. In this way values are calculated for  $\Delta_B^{(+)}(s)$

(5) M. Sugawara and A. Kanazawa, Phys. Rev. 123, 1895 (1962).

in the ranges  $22 \leq s \leq 32.7$  and  $59.6 \leq s \leq 80$ . These are shown in Fig. 2; the separate contributions are described below.

(a)  $\text{Re } B_0^{(+)}(s)$ . In the range  $59.6 \leq s \leq 80$ ,  $\text{Re } B_0^{(+)}(s)$  is evaluated in terms of Woolcock's s, p and d partial waves<sup>(6)</sup>, values being needed for energies up to about 215 MeV. In order to evaluate  $\text{Re } B_0^{(+)}(s)$  in the region  $s \leq (M-1)^2$ , use is made of the crossing relation

$$B^{\Theta}(s, t) = - B^{(+)}(u, t) , \quad (13)$$

where u is related to s and t by

$$u = 2M^2 + 2 - s - t , \quad (14)$$

together with (5), the definition of  $B_0^{(+)}(s)$ . For s in the range  $0 \leq s \leq (M-1)^2$  values of  $B^{(+)}(u, t)$  are only required for physical energies and angles<sup>(7)</sup>. In calculating  $\text{Re } B_0^{(+)}(s)$  for  $22 \leq s \leq 32.7$ ,  $\text{Re } B^{(+)}(u, t)$  is evaluated in terms of the s, p and d partial waves, values being needed for energies up to 400 MeV, and  $\text{Re } B_0^{(+)}(s)$  is then obtained by use of (13) and (5).

(b) The Physical Integral.  $\text{Im } B_0^{(+)}(s)$  is evaluated in terms of the s, p and d partial waves up to 400 MeV. Above 400 MeV contributions from the three resonances,  $T = \frac{1}{2}$  D 3/2 at 600 MeV,  $T = \frac{1}{2}$ , F 5/2 at 900 MeV and  $T = 3/2$ , F 7/2 at 1350 MeV are estimated from experimental data on total cross sections and inelasticity. Smooth background (i.e. non-resonant) terms are also added so as to fit onto the low energy values at 400 MeV and onto the alternative high

---

(6) See reference (1) for details of these partial waves.

(7) J. Hamilton, P. Menotti, T. D. Spearman and W. S. Woolcock Nuovo Cimento 20, 519 (1961)

---

energy behaviours at 2 BeV, one being set equal to zero above this energy and the others falling to zero as  $\pm 16 \pi / s^{1/2}$  above 2 BeV.

(c) The Crossed Integral: Here  $\text{Im } B_o^{(+)}(s)$  is evaluated using the crossing relation (13) in a similar manner to that described above for  $\text{Re } B_o^{(+)}(s)$ .

$\text{Im } B^{(+)}(u, t)$  is expressed in terms of the s, p and d waves below 400 MeV and by the three resonant terms, together with smooth background terms, in the region between 400 MeV and 2 BeV, enabling  $\text{Im } B_o^{(+)}(s)$  to be calculated for  $8 \leq s \leq 32.7$ . The behaviour as  $s \rightarrow 0$  is related to the behaviour of  $\text{Im } B^{(+)}(u, t)$  at high energies and backward angles<sup>(8)</sup>. Here again there is considerable uncertainty and two alternative forms for  $\text{Im } B_o^{(+)}(s)$  are calculated, one falling linearly to zero and the other remaining constant as  $s \rightarrow 0$ .

(d) The Born Term, This is evaluated using a value for the coupling constant  $G_F^2$ , corresponding to Woolcock's value for the pseudovector coupling constant,  $f^2 = 0.081$ <sup>(9)</sup>. The method of calculating the long range crossed Born term is similar to that described by Hamilton and Spearman<sup>(10)</sup>.

(e) Errors: These are of two types corresponding to errors on the low and high energy data. The errors associated with uncertainties in the high energy behaviour and also with the behaviour as  $s \rightarrow 0$  are hard to estimate. Some indications as to the form of these errors are provided by the alternative high energy behaviours considered. These errors, which are estimated to be

---

(8) J. Hamilton, T. D. Spearman and W. S. Woolcock, Ann. Phys. 17, 1 (1962).

(9) W. S. Woolcock, Proceedings of the Aix - en - Provence International Conference on Elementary Particles, Vol. I, 459. Also see reference (1).

(10) See appendix of reference (4) for details of the separation of long range crossed Born terms.

---

$\pm 5$  at  $s = 59.6$ , are only slowly varying functions of energy and it is especially important to note that it is very unlikely that they will produce a large displacement of the values of  $\Delta_B^{(+)}(s)$  for  $s \leq 32.7$  relative to those for  $s \geq 59.6$ .

The other type of error, due to uncertainties in the low energy data, has a stronger energy dependence. The physical and crossed integrals are dominated in this region by the  $(\frac{3}{2}, \frac{3}{2})$  resonance peak and so the main source of error lies in the values for  $\text{Re } B_0^{(+)}(s)$ . It is estimated that these errors are about  $\pm 1$  at the two thresholds, increasing slightly as  $s$  increases to 80 and increasing more rapidly as  $s$  decreases below  $s = 32.7$ , rising to  $\pm 2$  at  $s = 22$ . It should be noted that the errors in  $\text{Re } B_0^{(+)}(s)$  will satisfy at the thresholds, the crossing relation

$$\text{Error}(s = 59.6) = -\text{Error}(s = 32.7)$$

and that there will be a correlation of approximately this form away from the thresholds. Thus these errors will tend to displace the values of  $\Delta_B^{(+)}(s)$  for  $s \leq 32.7$  in the opposite direction to those for  $s \geq 59.6$ .

### 3. CONTRIBUTIONS FROM THE CIRCULAR CUT, $|s| = M^2 - 1$

#### (i) The Absorptive Part on the Circle.

The absorptive part of  $B^{(+)}(s, t)$  in the channel  $\pi\pi \rightarrow N + \bar{N}$  is given by the helicity amplitude expansion <sup>(11)</sup>

$$\text{Im } B^{(+)}(s, t) = 8\pi \sum_{J=2}^{\infty} \frac{J+1}{[J(J+1)]} q_3^J (ip_3 q_3)^{J-1} P_J'(\cos \theta_3) \text{Im } f_J(t), \quad (15)$$

even J

where

$$q_3^2 = t/4 - 1$$

$$p_3^2 = M^2 - t/4$$

$$\cos \theta_3 = (s - p_3^2 + q_3^2) / (2ip_3 q_3)$$

and  $\int_{-}^{\mathbf{J}}(t)$  are the helicity amplitudes. Ignoring those states with  $J \geq 4$  gives

$$\text{Im } B^{(+)}(s,t) = \frac{30\pi}{\sqrt{6}} (s+t/2 - M^2 - 1) \text{Im } f_-^2(t). \quad (16)$$

This expression then enables the discontinuity of  $B_o^{(+)}(s)$  across the circular cut to be calculated for that part of the circle having  $|\arg(s)| \leq 66^\circ$ , the series expansion (15) diverging beyond this arc.

The contribution to the discrepancy from a given arc around the front of the circle takes the form

$$\Delta_{B,\infty}^{(+)}(s) = \int_{\frac{1}{4}}^{t_{\max}} dt K(s,t) \text{Im } f_-^2(t), \quad (17)$$

when  $t_{\max}$  is related to  $\phi_{\max}$ , the maximum value of  $\arg(s)$  by

$$t_{\max} = 4 \left[ M^2 \sin^2(\phi_{\max}/2) + \cos(\phi_{\max}/2) \right].$$

The kernel  $K(s, t)$  can be calculated exactly<sup>(12)</sup> giving

$$K(s,t) = \frac{15}{\sqrt{6}} \left[ \frac{s-u}{4q_f^2} \ln \left( \frac{1+4q_f^2/t}{1+4q_f^2/t_{\max}} \right) - 2 \ln \left( \frac{t_{\max}}{t} \right) \right]. \quad (18)$$

(11) W. R. Frazer and J. R. Fulco, Phys. Rev. 119, 1420 (1960).

(12) See M. Marinaro and K. Tanaka, Nuovo Cimento 23, 537 (1962) for details of a similar calculation of the partial wave kernels.

(ii)  $\text{Im } f_-^2(t)$  and the  $J = 2, T = 0 \pi\text{-}\pi$  Interaction.

The Omnes method<sup>(13)</sup> is used to calculate the helicity amplitude in terms of a  $J = 2, T = 0 \pi\text{-}\pi$  phase shift  $\delta_2^0$ . The helicity amplitude  $f_-^2(t)$  is analytic in the  $t$  plane cut from  $4 \leq t < \infty$  and  $-\infty < t \leq a$ , where  $a = 4 - 1/M^2$ , and in addition has the phase  $\delta_2^0$  in the region  $4 \leq t \leq 16$ .

Consider the function

$$u(t) = \exp \left[ -\frac{1}{\pi} \int_4^\infty dt' \frac{\delta_2^0(t')}{t' - t} \right], \quad (19)$$

it being assumed that  $\delta_2^0(t')$  falls off sufficiently quickly for the integral to exist. Then, since  $u(t)$  is real for  $t \leq 4$  and has the phase  $-\delta_2^0(t)$  along the cut  $4 \leq t < \infty$ ,  $u(t) \cdot f_-^2(t)$  has only the cuts  $-\infty < t \leq a$  and  $16 \leq t < \infty$ .

In the region  $-25 \leq t \leq a$ , values for  $\text{Im } f_-^2(t)$  are calculated in terms of the single nucleon Born term and the  $\pi\text{-}N (3/2, 3/2)$  partial wave amplitude<sup>(14)</sup>, the results being shown in Fig. 3. The contributions of the other  $\pi\text{-}N$  partial waves can be neglected since they are much smaller than the  $(3/2, 3/2)$  term which is itself only 34% of the Born term at  $t = -25$ .

Beyond  $t = -25$  the series expansion of  $\text{Im } f_-^2(t)$  in terms of  $\pi\text{-}N$  partial waves diverges and values for  $\text{Im } f_-^2(t)$  cannot be calculated in this region.

Accordingly  $f_-^2(t)$  is given by

$$f_-^2(t) = \frac{1}{u(t)} \left[ -\frac{1}{\pi} \int_{-25}^a dt' \frac{u(t') \text{Im } f_-^2(t')}{t' - t} + \frac{C}{t + t_0} \right], \quad (20)$$

(13) R. Omnes, Nuovo Cimento 8, 316 (1958). Also see reference (14).

(14) W. R. Frazer and J. R. Fulco, Phys. Rev. 117 1603 (1960).

where the pole term has been added to approximate the contribution of the region  $-\infty < t \leq -25$  and where the additional contribution above the four pion threshold has been neglected.

The values of the pole position and the residue are determined by fitting (20) and the corresponding derivative relation to values for  $\text{Re } f_-^2(0)$  and  $\frac{d}{dt} \text{Re } f_-^2(t)|_{t=0}$ . These constants have been calculated by the method of Ball and Wong<sup>(15)</sup>, using forward direction  $\pi$ -N scattering data, giving the values

$$\text{Re } f_-^2(0) = -0.260 \pm 0.006 \quad (21)$$

$$\text{Re } f_-^{2'}(0) = -0.069 \pm 0.002 \quad (22)$$

The errors are due primarily to the error on the value of  $f_-^2$ , the total value of the other contributions being only about 2% of the Born term.

In order to calculate values for the helicity amplitude,  $f_-^2(t)$ , it is convenient to introduce some parametric representation for the phase  $\delta_2^\circ(t)$ . A suitable two parameter form having the correct threshold behaviour and giving a phase which rises to a single maximum and then falls to zero at high energies is

$$\delta_2^\circ(t) = \frac{aq_3^5}{1 + bq_3^6}, \quad q_3 \geq 0. \quad (23)$$

(15) J. S. Ball and D. Y. Wong, Phys. Rev. Letters 6, 29 (1961).

This then enables  $u(t)$  to be evaluated exactly giving

$$u(t) = e^{-i\delta_2^0} \exp \left\{ \frac{-a}{1+bq_3^6} \left[ \frac{2}{3} \left(\frac{1}{b}\right)^{5/6} + q_3^2 \frac{1}{3} \left(\frac{1}{b}\right)^{1/2} + q_3^4 \frac{2}{3} \left(\frac{1}{b}\right)^{1/6} \right] \right\} \quad t \geq 4$$

$$= \exp \left\{ \frac{-a}{1+bq_3^6} \left[ \frac{2}{3} \left(\frac{1}{b}\right)^{5/6} + q_3^2 \frac{1}{3} \left(\frac{1}{b}\right)^{1/2} + q_3^4 \frac{2}{3} \left(\frac{1}{b}\right)^{1/6} - (-q_3) \right] \right\} \quad t < 4 \quad (24)$$

In this way  $\text{Im } f_-(t)$  can be calculated in the region  $t \geq 4$  for any values of the parameters  $a$  and  $b$ . Substitution in (17) then gives the contribution from the front of the circle to  $\Delta_B^{(+)}(s)$  for the particular phase  $\delta_2^0(t)$  chosen.

#### 4. RESULTS.

The discrepancy  $\Delta_B^{(+)}(s)$  is shown in Fig. 2. The value is only about 15% of that of the low energy values for  $\text{Re } B_0^{(+)}(s)$  and it can be seen that it is a very slowly varying function of energy over the range  $22 \leq s \leq 80$ . There is slight curvature near to the two thresholds but this is very small and can be completely removed by small variations of the low energy p-wave contributions.

It should first be noted that the discrepancy is well fitted by a simple pole situated on the left hand cut as can be seen in Fig. 4 (i). Here the errors shown are only those due to uncertainties in the values of  $\text{Re } B_0^{(+)}(s)$  which have the correlation noted in Sec. 2 (e). In judging this and subsequent fits it should be remembered that if the values of the discrepancy in the region  $s \geq 59.6$  are increased by changes in the values of  $\text{Re } B_0^{(+)}(s)$  then the values in the region  $s \leq 32.7$  are decreased or vice versa.

In view of certain evidence that the  $J = 2, T = 0 \pi\pi$  interaction may be fairly strong, it is of interest to obtain an upper bound on the phase

shift  $\delta_2^o$  consistent with this discrepancy. The work of Atkinson<sup>(16)</sup> gave values for  $\delta_2^o$  rising to around  $45^\circ$  at  $t = 21$  while Lovelace and Masson<sup>(17)</sup> obtained values rising to  $50^\circ \pm 10^\circ$  at  $t = 30$ . Accordingly, values of the parameters  $a$  and  $b$  are chosen so as to give a phase with maximum value around  $t = 22$  and the contribution of  $\text{Im } f_-^2$  over the front of the circle calculated. The best fit to the discrepancy is then obtained by adding a pole term to represent the remaining contributions. The results are shown in Fig. 4 (ii) where it can be seen that it is impossible to fit the shape of the discrepancy if the maximum value of the phase,  $\delta_2^o$ , rises above  $13^\circ$  at  $t = 22$ . If the parameters are altered so as to move the peak out to  $t = 36$  then the maximum value of the phase giving an acceptable fit increases, a maximum value of  $18^\circ$  giving a fit well within the errors. These different phases are shown in Fig. 5. It should be noted that these results will be insensitive to all but the strangest high energy behaviour since changes in the discrepancy due to changes in the high energy terms should only alter the pole terms which have been added to represent the effect of distant singularities.

It has also been suggested, both theoretically<sup>(18)</sup> and experimentally<sup>(19)</sup> that the phase,  $\delta_2^o$ , may resonate around 1200 MeV. A meaningful calculation of the contribution of such a phase to the discrepancy is very much more difficult. A helicity amplitude obtained by solving (20) with a phase which

(16) See reference (2) for details of the phase shift.

(17) C. Lovelace and D. Masson, Proceedings of the 1962 International Conference on High Energy Physics, CERN, p. 510.

(18) S. D. Drell, ibid. p. 906.

(19) J. Henneny, J.J. Veillet, M. di Corato & P. Negri, ibid. p. 603. Also see J.J. Veillet, J. Hennessy, H. Bingham, M. Block, D. Drigard, A. Lagarrique, P. Mittner, A. Rousset, G. Bellini, M. Di Corato, E. Fiorrini & P. Negri. Phys. Rev. Letters 10, 29 (1962).

is large in the high energy region is subject to large errors due to neglect of the inelastic contributions and to the increasing importance of the errors <sup>Also the main contribution of this amplitude to the absorptive part of  $B^+$ )</sup> on the pole position and residue <sup>(20)</sup> around the circle will occur beyond the region of convergence of the helicity amplitude expansion.

If such a d wave resonance is sufficiently narrow for the corresponding helicity amplitude to be neglected in the low energy region it is possible to approximate  $\text{Im } f_-^2(t)$  by a single  $\delta$ -function. If it is further assumed that (16) represents an asymptotic expression for the absorptive part of  $B^{(+)}$ , even beyond the region of convergence, then the contribution of such a  $\delta$ -function approximation over the whole of the circle can be calculated. The fit using such an approximation, together with a pole to represent the left hand cut, is shown in Fig. 4 (iii). It can be seen that the fit is quite good, but it is impossible to say whether the normalisation constant associated with the  $\delta$ -function is consistent with a resonant phase or only with a fairly sharp peak in the high energy values of  $\text{Im } f_-^2(t)$ .

Thus the  $\pi^- N$  scattering data is consistent with a  $J = 2, T = 0$   $\pi-\pi$  phase,  $\delta_2^0$ , which does not rise above a maximum value of about  $13^\circ$  at  $t = 22$  (650 Mev) or  $18^\circ$  at  $t = 36$  (840 MeV). The data is also consistent with a  $\delta$ -function contribution at  $t = 70$  (1170 Mev) but the difficulties involved in solving the Omnes equation at these high energies make it impossible to say whether the normalisation constant associated with this  $\delta$ -function is consistent with a phase shift having a fairly narrow resonance or whether it only corresponds to a peak in the absorptive part of the helicity amplitude  $f_-^2(t)$ .

---

(20) See L.L.J. Vick, U.C.L. Physics Dept. preprint 1963 for a discussion of the difficulties associated with solving the Omnes equation at high energies.

ACKNOWLEDGMENTS.

I would like to thank Professor J. Hamilton for suggesting this problem and also for his helpful discussions and criticism of this work. I would also like to express my gratitude to London University Scholarships Committee for a Studentship.

FIGURE CAPTIONS

Fig. 1. The singularities of the amplitude  $B_0^{(+)}$  in the complex s-plane.

Fig. 2. Values for  $\text{Re } B_0^{(+)}$  and for the discrepancy  $\Delta_B^{(+)}(s)$ .

The vertical lines indicate the changes in  $\Delta_B^{(+)}(s)$  produced by the different high energy behaviours.

Fig. 3. Values of  $\text{Im } \int_{-25}^a f(t) dt$  for  $-25 \leq t \leq a$ . The broken curve indicates the Born term contribution.

Fig. 4. Fits to the discrepancy. The vertical lines represent the estimated errors at threshold due to uncertainties in the low energy data.

(i) represents the fit by a single pole on the left hand cut; (ii) gives the fits for various phase shifts  $\delta_2^0$ . (iii) gives the fit for a  $\delta$ -function contribution at  $t = 70$ . In comparing the fits it is important to note the approximate correlation of errors described in Sec. 2 (iv).

Fig. 5. Values of the phase,  $\delta_2^0$ , giving acceptable fits to the discrepancy.

FIG. 1.

COMPLEX S-PLANE

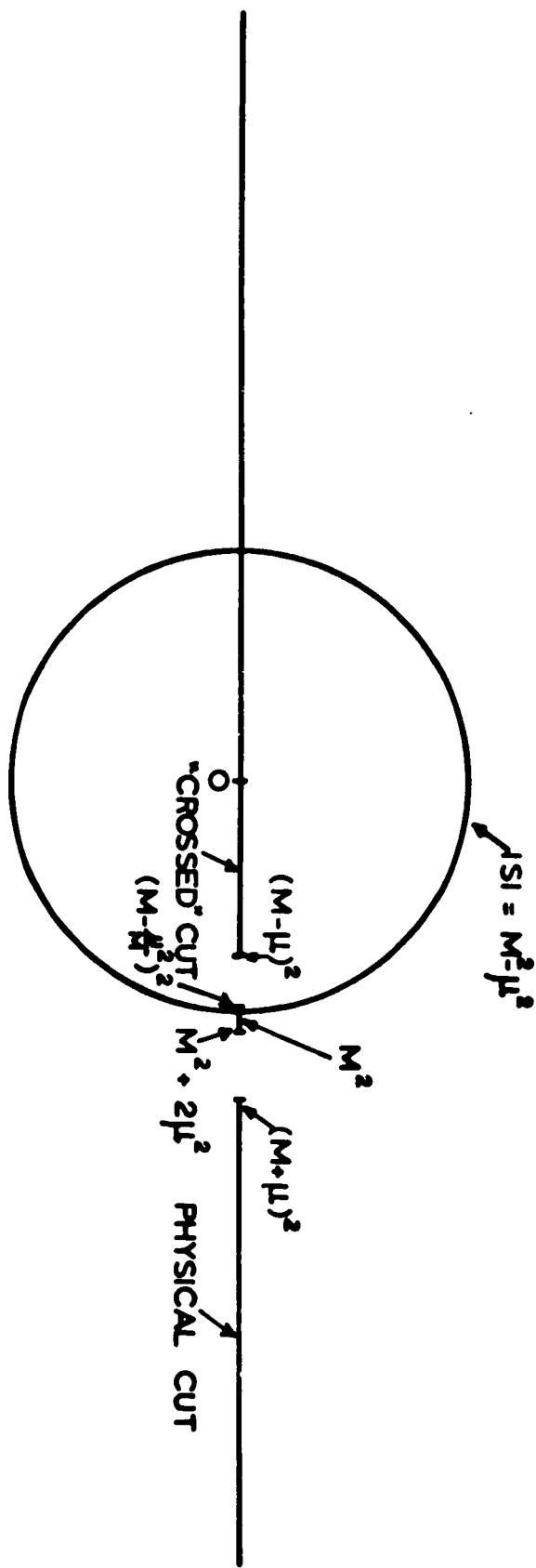


Fig.2.

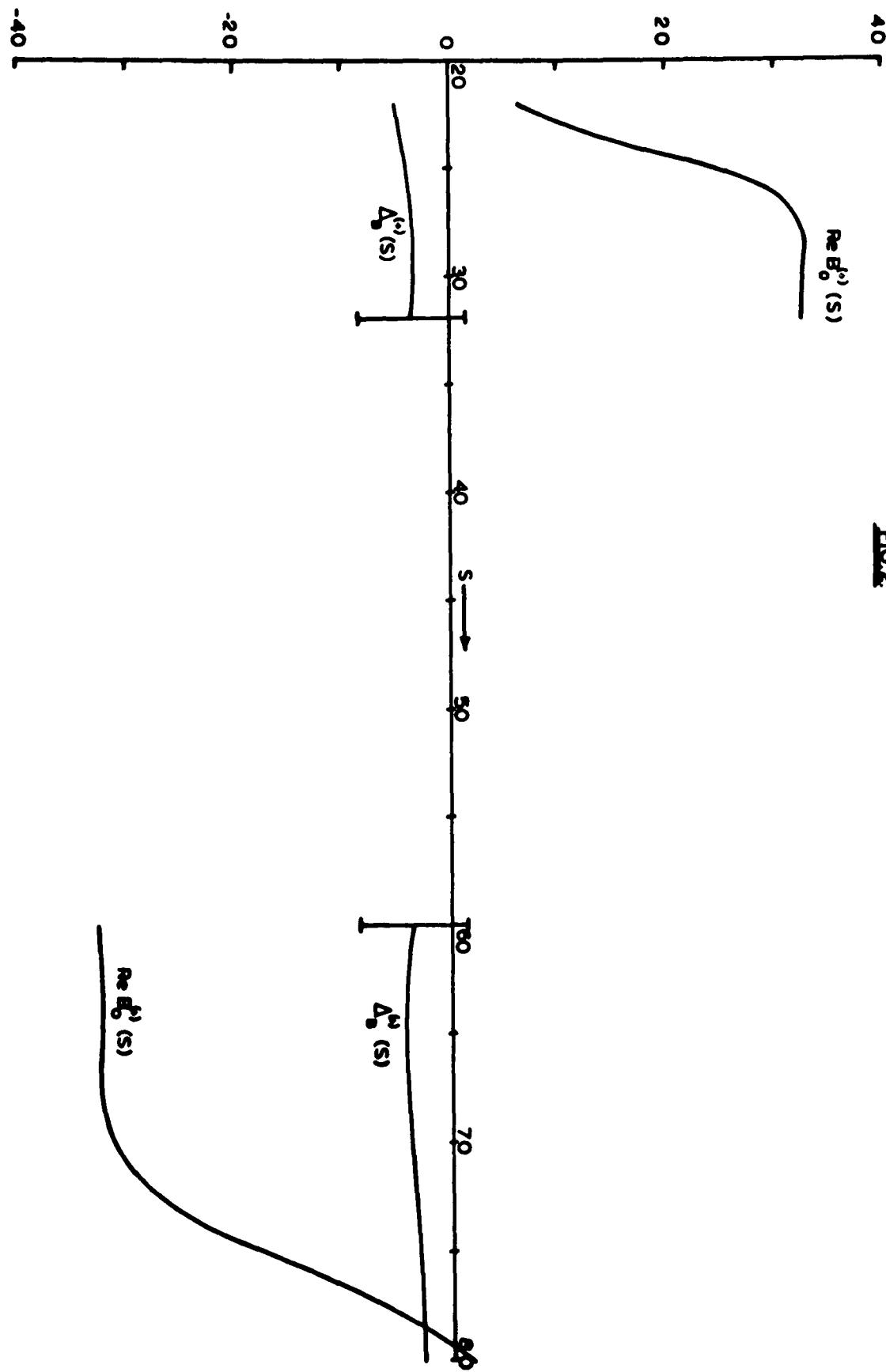


FIG. 3.

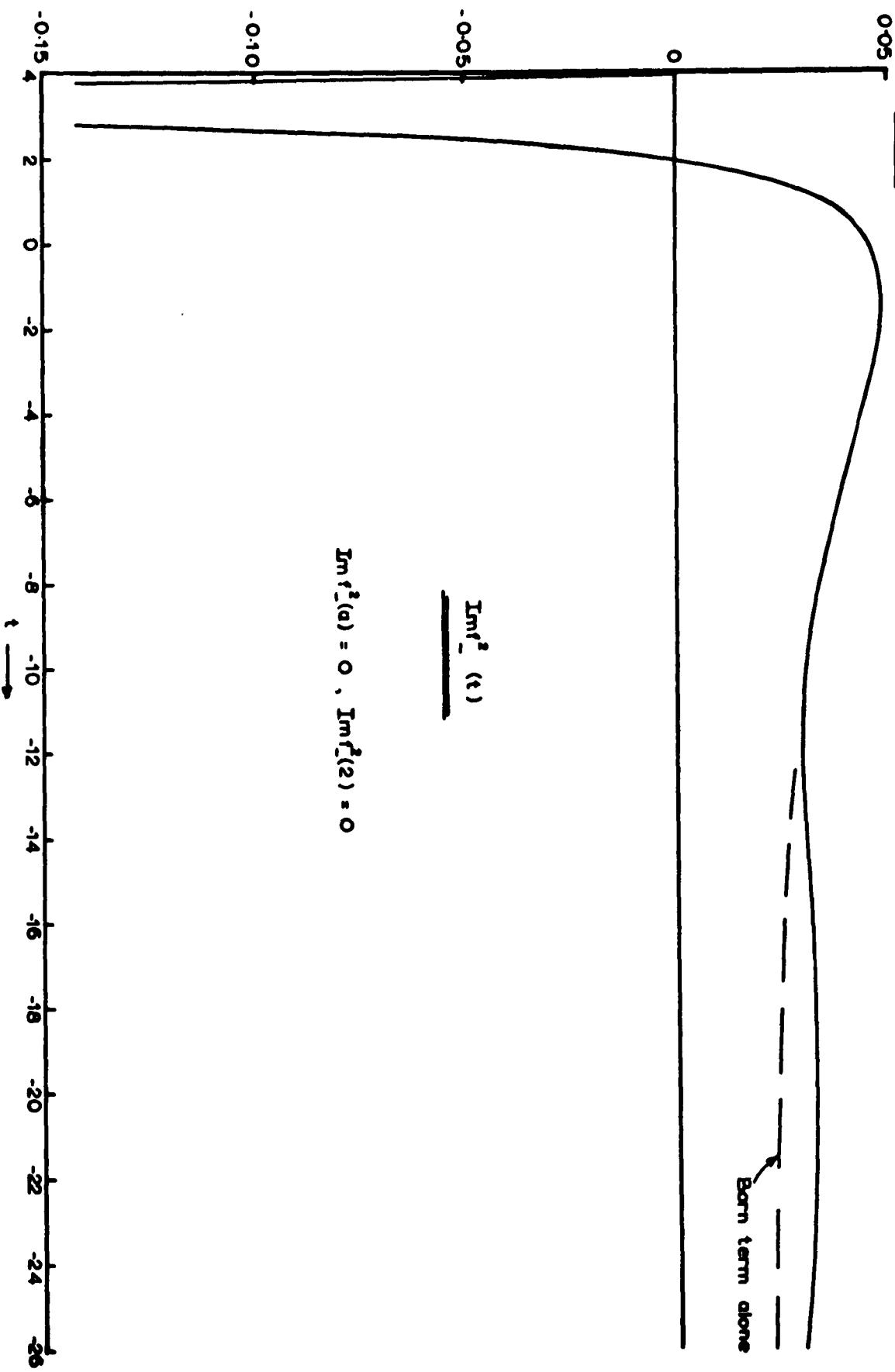


FIG. 4 (i)

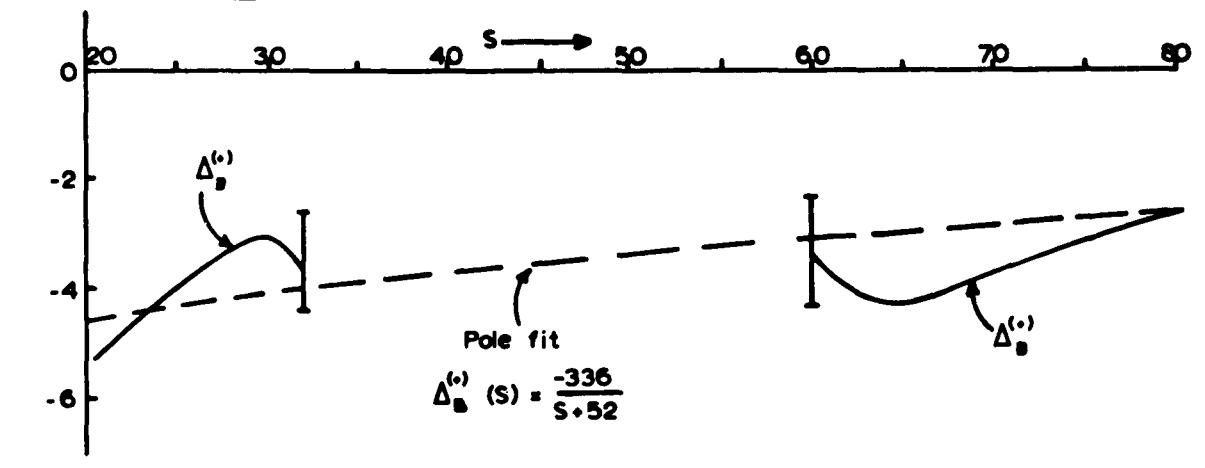


FIG. 4 (ii)

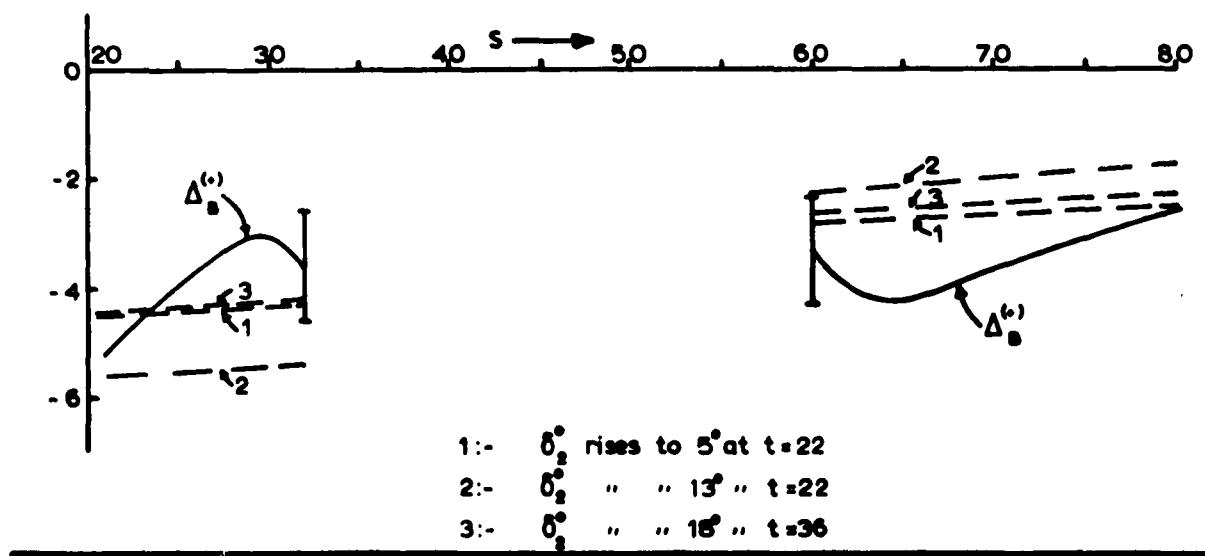


FIG. 4 (iii)

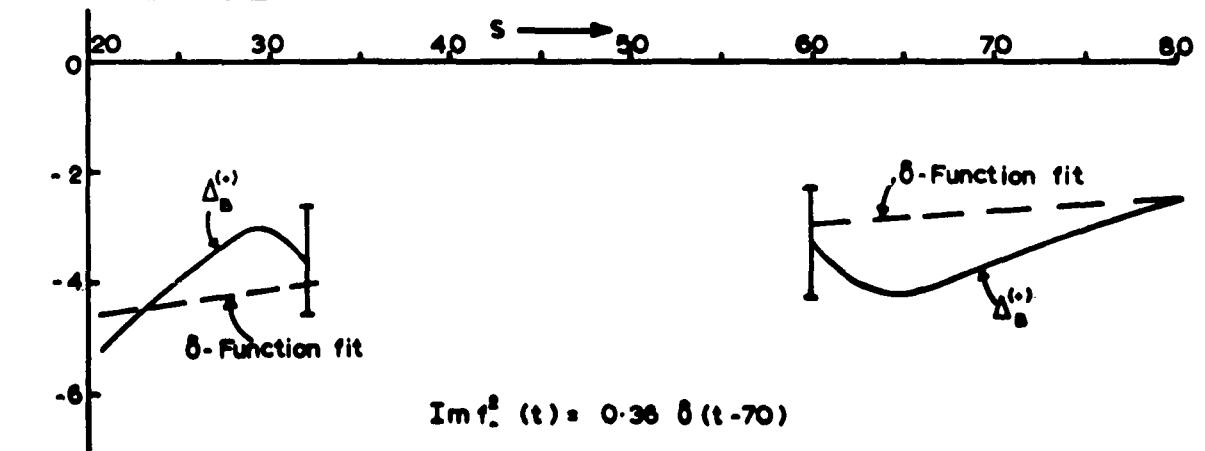
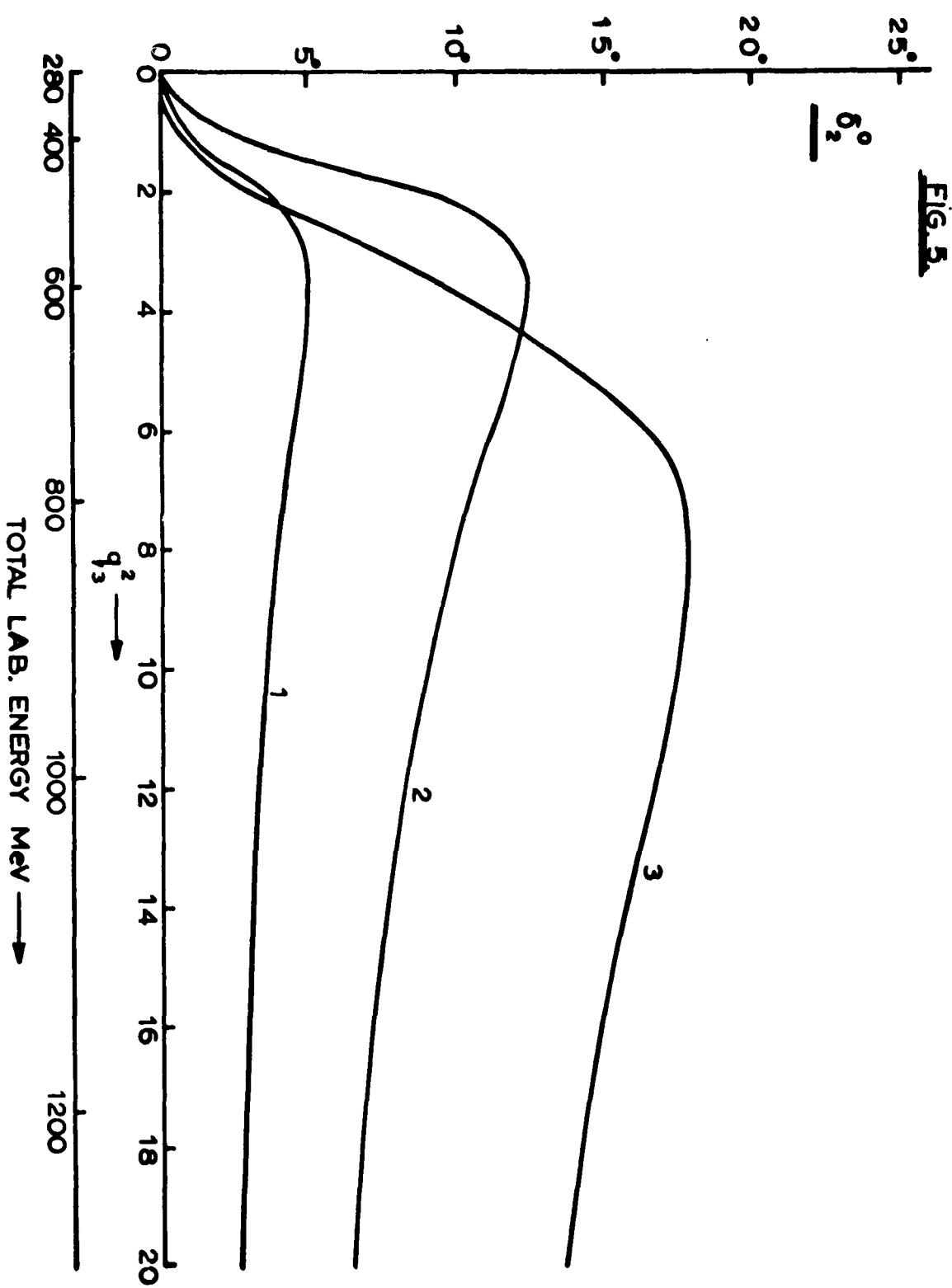


FIG. 5.



<p>Department of Physics University College Gower Street London W.C.1.</p>	<p>AF EOAR Grant 63-3 TN - 13 PHYSICS</p>	<p>Department of Physics University College Gower Street London W.C.1.</p>	<p>AF EOAR Grant 63-3 TN - 13 PHYSICS</p>
<p>Technical Note No. 13 20th May, 1963</p>	<p>Pion-Nucleon Scattering and the J=2, T=0 Pion-Pion Interaction</p>	<p>G. C. Oades</p>	<p><u>ABSTRACT</u> The effect of the J=2, T=0 <math>\pi\pi</math>-interaction on the <math>\pi</math>-N invariant amplitude <math>B^{\pi}</math> is analysed. The data is inconsistent with a phase <math>\delta_2^{\circ}</math> which rises above <math>130^\circ</math> around 650 MeV but is consistent with a <math>\delta</math>-function contribution at 1200 MeV.</p>
<p>Department of Physics University College Gower Street London W.C.1.</p>	<p>AF EOAR Grant 63-3 TN - 13 PHYSICS</p>	<p>Department of Physics University College Gower Street London W.C.1.</p>	<p>AF EOAR Grant 63-3 TN - 13 PHYSICS</p>
<p>Technical Note No. 13 20th May, 1963</p>	<p>Pion-Nucleon Scattering and the J=2, T=0 Pion-Pion Interaction</p>	<p>G. C. Oades</p>	<p><u>ABSTRACT</u> The effect of the J=2, T=0 <math>\pi\pi</math>-interaction on the <math>\pi</math>-N invariant amplitude <math>B^{\pi}</math> is analysed. The data is inconsistent with a phase <math>\delta_2^{\circ}</math> which rises above <math>130^\circ</math> around 650 MeV but is consistent with a <math>\delta</math>-function contribution at 1200 MeV.</p>
<p>Department of Physics University College Gower Street London W.C.1.</p>	<p>AF EOAR Grant 63-3 TN - 13 PHYSICS</p>	<p>Department of Physics University College Gower Street London W.C.1.</p>	<p>AF EOAR Grant 63-3 TN - 13 PHYSICS</p>
<p>Technical Note No. 13 20th May, 1963</p>	<p>Pion-Nucleon Scattering and the J=2, T=0 Pion-Pion Interaction</p>	<p>G. C. Oades</p>	<p><u>ABSTRACT</u> The effect of the J=2, T=0 <math>\pi\pi</math>-interaction on the <math>\pi</math>-N invariant amplitude <math>B^{\pi}</math> is analysed. The data is inconsistent with a phase <math>\delta_2^{\circ}</math> which rises above <math>130^\circ</math> around 650 MeV but is consistent with a <math>\delta</math>-function contribution at 1200 MeV.</p>